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Linear Stability of the Compressible Reacting Mixing Layer

D. S. Shin* and J. H. Ferziger†
Stanford University, Stanford, California 94305

This paper investigates the linear stability of mixing layers with special emphasis on the effects of heat release and compressibility. The results show that multiple supersonic modes exist for both nonreacting and reacting flows when the disturbance phase velocity is supersonic relative to the freestream. These supersonic modes become less unstable with increasing Mach number but more unstable with increasing heat release. The most unstable supersonic modes are three-dimensional (oblique) for nonreacting flows but two-dimensional for reacting flows. The structure of the disturbed flows suggests that the supersonic modes do not enhance mixing between the fuel and oxidizer. Finally, the convective Mach number does not appear to be a universal parameter for characterizing compressibility in reacting flows.

I. Introduction

REACTING free shear layers occur in many systems, including gas turbine combustors and rockets. Chemical reaction can occur only when the reactants are molecularly mixed. However, short residence times require efficient mixing between the fuel and oxidizer. This is especially important in air-breathing ramjets. Fast mixing requires the flow to be vigorously turbulent, which requires the laminar flow to be unstable. Hence, understanding of the stability characteristics of reacting free shear layers may lead to techniques for enhancing mixing or controlling the flow. Stability analysis can also predict some characteristics of the turbulent reacting mixing layer. The conclusions may be expected to apply, with quantitative modifications, to other shear flows, e.g., jets.

Stability of the compressible mixing layer has received less study than the incompressible flow. Gropengiesser[1] carried out inviscid spatial-stability calculations for the compressible mixing layer and found that compressibility stabilizes the flow. He also found that the most unstable modes are three-dimensional at high Mach numbers. Sandham and Reynolds[2] solved the linearized inviscid compressible stability equation and found multiple unstable modes. They also found that three-dimensional effects are important at high Mach numbers, which was confirmed experimentally by Clemens and Mungal.[3] Jackson and Grosch[4] carried out inviscid spatial calculations and found two sets of unstable modes if the Mach number exceeds a critical value. They also studied the effect of heat release on the spatial stability of supersonic reacting mixing layers using analytical velocity profiles and the flame-sheet concept.[5] They found that sufficient heat release could make the flow absolutely unstable. In the subsonic case, Mahalingam et al.[6] found that heat release stabilizes coflowing chemically reacting jets. Shin and Ferziger[7] showed that in the low-Mach-number reacting mixing layer, the stability properties are sensitive to the mean flow profiles, so it is important to use the correct laminar profiles in the stability analysis. Outer modes are found in low-speed flows when the heat release is significant and become dominant at large heat release.

This paper considers a plane mixing layer in which the fuel and oxidizer are initially unmixed. The chemistry is finite-rate, single-step irreversible reaction with Arrhenius kinetics. Since earlier work[7] showed the importance of using correct laminar profiles rather than assumed analytical ones, laminar profiles obtained by solving the compressible boundary-layer equations are used as inputs to the linear-stability analysis. To reduce the parameter space, we considered the cases in which the ratio of the speed of the slow stream to that of the fast stream is 0.5 and only spatially developing layers. We use nondimensional adiabatic flame temperature T_{ad} to express the amount of heat release from combustion. For a given T_{ad}, the actual temperature rise in high-Mach-number flows may be higher than in low-Mach-number flows due to viscous dissipation. How best to express the temperature rise from combustion and viscous dissipation in a single parameter is still an open question. Finally, we consider only convectively unstable cases, as in the previous paper.[7]

II. Linear Disturbance Equation

In free shear flows, viscosity damps growing disturbances. Even though the large values of heat release in reacting flows will increase the viscosity, the inviscid analysis of low-speed reacting mixing layers showed good agreement with experiments and numerical simulations.[7,8] Thus, the simpler inviscid stability problem, which yields an upper bound on the growth rate, will be considered exclusively in this study. To simplify the stability analysis, we assume that the laminar flow is parallel, i.e., that its variation is entirely in the direction normal to the flow. The mean pressure is assumed constant. All variables are taken to be sums of their laminar values and small perturbations of traveling wave form. Thus, all flow variables can be represented as

$$f(x, y, z, t) = \bar{f}(y) + f'(x, y, z, t) \tag{1}$$

$$f'(x, y, z, t) = \hat{f}(y) e^{i(\alpha x + \beta z - \omega t)} \tag{2}$$

where $\bar{f}(y)$ is the laminar profile of a quantity, $\hat{f}$ depends only on the y coordinate, and α and β are wave numbers in the streamwise (x) and spanwise (z) directions, respectively, and ω is the frequency. All dependent variables are normalized by their values on the high-speed side (subscript 1). The relation between the wave numbers and the propagation angle of disturbance relative to the x direction is

$$\tan \theta = \beta / \alpha_r \tag{3}$$

where α_r is the real part of α. Since the disturbances do not grow in z, the wave number β is real. Because a spatial

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*Graduate Research Assistant, Thermosciences Division, Department of Mechanical Engineering; currently Visiting Researcher, Mechanical Engineering Laboratory, Daikin Industries, Ltd., 1304, Kanaoka-cho, Sakai, Osaka 591, Japan.

†Professor, Thermosciences Division, Department of Mechanical Engineering. Member AIAA.

problem is considered here, ω is real and α is allowed to be complex. The amplification rate is $-\alpha_i$.

The perturbation equations are derived by linearizing the Euler equations, which are obtained by dropping the diffusion terms from the Navier-Stokes equations. Substituting Eq. (2) into these equations and neglecting the products of disturbances yields the disturbance equation for the pressure:

$$\hat{p}'' - \left\{\frac{2\alpha\bar{u}'}{(\alpha\bar{u}-\omega)} + \frac{\bar{\rho}}{\bar{T}}(\alpha\bar{u}-\omega)^2 [RXN1]\right\}\hat{p}' - \left[(\alpha^2+\beta^2) - \gamma M_1^2(\alpha\bar{u}-\omega)^2\left\{\frac{1}{\bar{T}} + \frac{\bar{\rho}}{\bar{T}}[RXN2]\right\}\right]\hat{p} = 0 \quad (4)$$

where $\bar{u}$, $\bar{\rho}$, and $\bar{T}$ are the mean velocity, density, and temperature, respectively; and a prime denotes differentiation with respect to y. Here, γ is the specific heat ratio and M_1 is the Mach number of the upper stream. The terms $[RXN1]$ and $[RXN2]$ represent the effect of density variation due to chemical reaction and compressibility on the instability; they are given in the Appendix. Note that Eq. (4) is a homogeneous second-order ordinary differential equation.

Equation (4) requires two boundary conditions. These require the pressure perturbation to be bounded as $|y| \to \infty$. This can be made more precise by considering the asymptotic form of the solutions of Eq. (4). As $y \to \pm\infty$, $\bar{u}'$ and $[RXN1]$ become negligible and $[RXN2]$ becomes $-\bar{T}(\gamma-1)/\gamma$. Equation (4) then reduces to

$$\hat{p}'' - q^2\hat{p} = 0 \quad (5)$$

where

$$q^2 = (\alpha^2+\beta^2) - \frac{M_1^2}{\bar{T}}(\alpha\bar{u}-\omega)^2 \quad (6)$$

Note that q^2 can be positive, negative, or complex. This will play an important role below. Far from the shear layer, the pressure must behave like

$$\hat{p} \to \exp(\pm qy) \qquad \text{as } y \to \mp\infty \quad (7)$$

The asymptotic behavior of the disturbances can be inferred from this result. The nature of the disturbance can be described in terms of the relative Mach numbers, M_{ri} $(i = 1, 2)$, which are defined as the Mach numbers of the disturbances in the direction of the wavevector (α, β) relative to freestreams i ($i = 1$ denotes the upper stream and $i = 2$, the lower stream):

$$M_{r1} = \frac{\alpha M_1(c-1)}{(\alpha^2+\beta^2)^{1/2}}, \qquad M_{r2} = \frac{\alpha M_1(c-\bar{u}_2)}{(\alpha^2+\beta^2)^{1/2}\,\bar{T}_2^{1/2}} \quad (8)$$

where c is the phase velocity of the disturbance. When the magnitude of a relative Mach number is less than unity, the instability wave is said to be subsonic with respect to that boundary; when it is greater than unity, it is said to be supersonic with respect to that boundary. At the zeros of q^2, the instability waves are sonic ($|M_r| = 1$). We define c_u as the phase speed of a disturbance that is sonic with respect to the upper stream and c_l as the corresponding speed with respect to the lower stream:

$$c_u = 1 - \frac{(\alpha^2+\beta^2)^{1/2}}{\alpha M_1}, \qquad c_l = \bar{u}_2 + \frac{(\alpha^2+\beta^2)^{1/2}\,\bar{T}_2^{1/2}}{\alpha M_1} \quad (9)$$

The relative Mach number of the most unstable mode was called the convective Mach number by Mack.[9] The quantity q is real only for neutral subsonic disturbances; for neutral supersonic disturbances it is purely imaginary; in all other cases it is complex. When the disturbances are subsonic with respect to either freestream, Eq. (7) must be satisfied so that the disturbances remain bounded at infinity. These modes are similar to the ones arising in incompressible stability theory. For supersonic disturbances, the instability waves are oscillatory as $|y| \to \infty$ and move along the constant phase lines

$$\alpha_r x + \beta_r z \pm q_i y - \omega t = 0 \quad (10)$$

Supersonic neutral disturbances may be classified as incoming ($q_i > 0$) or outgoing ($q_i < 0$) for the upper freestream; the signs are reversed for the lower freestream. Only outgoing waves are acceptable; incoming waves must be rejected. Setting the amplitudes of the incoming waves to zero provides the required boundary conditions.

An iterative method based on the shooting and Newton-Raphson methods is used to solve Eq. (4). To begin, a guess at the eigenvalue is made. For a spatial analysis, ω is specified and α is guessed. Given the eigenvalue, $\hat{p}$ for large y is given by Eq. (7). The computation starts at some large value of y on each side; the starting value of y must be within the range of y for which laminar flow is tabulated but large enough for the solutions to be essentially constant. Then we integrate Eq. (4) from both freestreams toward the centerline of the mixing layer. We used the subroutine ODE[10] for the integrations with an error control parameter of 10^{-7}. At the centerline, $y = 0$, the values of $\hat{p}$ and $\hat{p}'$ computed by integration from the upper stream, $\hat{p}_+(0)$ and $\hat{p}_+'(0)$, are compared with the values computed by integration from the lower stream, $\hat{p}_-(0)$ and $\hat{p}_-'(0)$. If they match, the process has converged; if not, a new eigenvalue is chosen by the Newton-Raphson method and iteration continues until the eigenvalue increment is reduced to 10^{-6}. We used double-precision arithmetic, which allowed computation of even weakly amplified unstable modes at high Mach numbers.

III. Results

A. Possibility of Noninflectional Supersonic Modes

In compressible flows, the necessary and sufficient condition for the existence of a neutral subsonic wave is $(\bar{u}'/\bar{T})' = 0$. The location of the zero is called the generalized inflection point. The proof of sufficiency given by Lees and Lin[11] requires the relative Mach number to be subsonic at the generalized inflection point. We derived a similar criterion for low-Mach-number reacting flows and found that nonreacting incompressible mixing layers have an inflection point, whereas reacting mixing layers with sufficient heat release have three.[7]

The existence of multiple unstable modes at supersonic relative Mach numbers was first discovered in the extensive numerical work of Mack.[12] At about the same time, Gill[13] found multiple solutions in his study of "top hat" jets and wakes. The modes that they found do not require a generalized inflection point. The reason for these multiple modes can be understood by examining the two-dimensional inviscid stability equation for the pressure disturbance, Eq. (5). Using the definition of the relative Mach number, M_r [Eq. (8)], Eq. (5) can be written

$$\hat{p}'' - \alpha^2(1-M_r^2)\hat{p} = 0 \quad (11)$$

Using Eq. (2), we may rewrite Eq. (11) as a partial differential equation

$$\frac{\partial^2 p'}{\partial y^2} + (1-M_r^2)\frac{\partial^2 p'}{\partial x^2} = 0 \quad (12)$$

where p' is the pressure disturbance. When $M_r^2 < 1$, Eq. (12) is elliptic, and the unique solution is connected with the generalized inflection point.[10] However, when $M_r^2 > 1$, Eq. (12) becomes hyperbolic, and many solutions can satisfy the boundary conditions at $y = \pm\infty$. Mack[9] found similar behavior in compressible boundary layers.

Like nonreacting compressible boundary layers,[9] nonreacting mixing layers have one generalized inflection point even at high Mach numbers. Figure 1 shows that the nonreacting mixing layer at $M_1 = 5$ ($M_2 = 2.5$) has one generalized inflection point, whereas the reacting mixing layer at the same Mach number with $T_{ad} = 4$ has three such points as does the low-speed reacting flow.[7] In this case, M_c, the convective Mach number defined by Papamoschou and Roshko,[14] is 1.25. The convective Mach number will be discussed further below. Even though the nonreacting mixing layer has just one generalized inflection point, multiple unstable modes may exist when a region of the flow is supersonic relative to the disturbance phase velocity. In other words, supersonic unstable modes can exist in the absence of a generalized inflection point.

B. Effect of Mach Number and Heat Release

Figure 2 shows the amplification rates and phase speeds as functions of frequency for $M_1 = 5$ ($M_c = 1.25$). The nonreacting mixing layer ($T_{ad} = 1$) has two unstable modes even though it has but one generalized inflection point. One is supersonic relative to the lower freestream and the other is supersonic relative to the upper freestream. We call the former the fast mode and the latter the slow mode in accord with their phase speeds. Each has nearly the same growth rate but the fast mode is more broadband. Neither resembles the vorticity modes of boundary layers and low-speed reacting mixing layers, which require generalized inflection points to be unstable, and are, of course, subsonic. Experiments[15-17] have observed the slow and fast modes that have lower and higher propagation velocities than the mean convection velocities, although we may not compare turbulent experiments with the linear analysis directly.

The reacting mixing layer ($T_{ad} = 4$) also has two supersonic unstable modes, even though it has three generalized inflection points in Fig. 1. They are outer modes and their asymptotic phase speeds shown in Fig. 2b are very close to the laminar velocities at the outer generalized inflection points in Fig. 1, even though the mean velocities at the generalized inflection points are supersonic relative to one of the freestream velocities. This suggests that the instability modes in reacting flows may be continuations of the inflectional modes of low-Mach-number flows. The outer modes have twice the growth rate of the corresponding modes of the nonreacting flow; the fast mode is slightly less amplified and more broadband than the slow mode. Figure 2b shows that the phase velocity of the fast mode approaches an asymptotic value close to the upper freestream speed as the frequency increases; the slow mode phase speed asymptotes to the speed of the lower freestream. Increasing the adiabatic flame temperature raises the speed of the fast mode and reduces the speed of the slow mode, as in low-speed reacting mixing layers.[7]

To study the effect of Mach number on the type of disturbances produced, we give the phase speeds of the most unstable modes, c_r, as functions of Mach number in Fig. 3 for

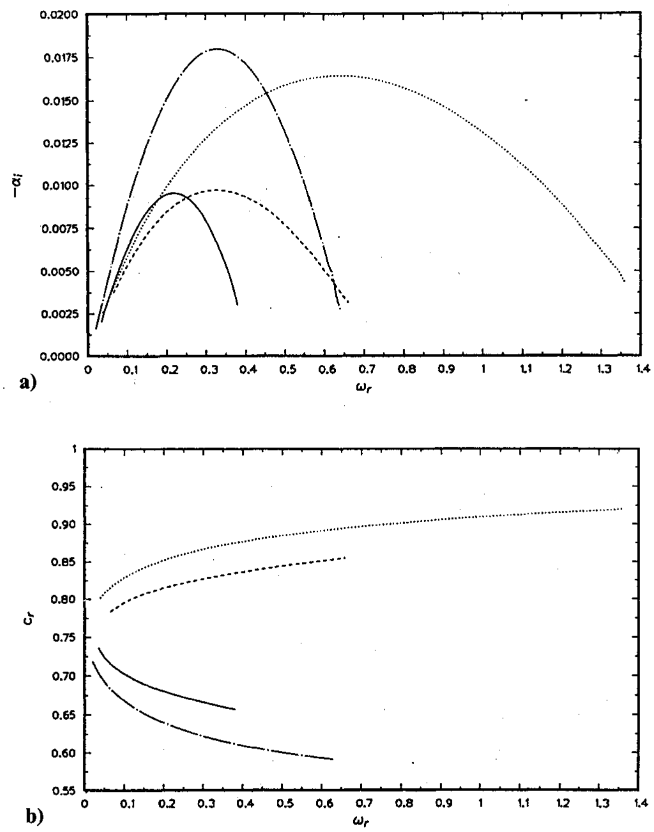

Fig. 2 The multiple instability modes in the compressible flow: a) growth rate, and b) phase velocity. $M_1 = 5$, $\bar{T}_2 = 1$, $\beta = 0$. ——, slow mode ($T_{ad} = 1$); ----, fast mode ($T_{ad} = 1$); — · —, slow mode ($T_{ad} = 4$); · · · · · ·, fast mode ($T_{ad} = 4$).

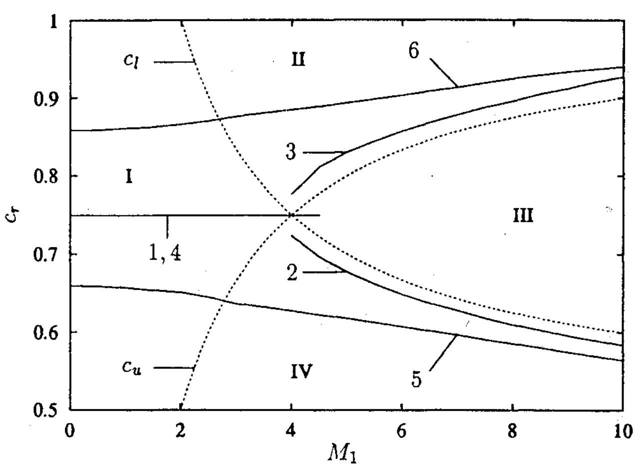

Fig. 3 Phase speeds of the most unstable modes vs Mach number. $\bar{T}_2 = 1$, $\beta = 0$. 1, center mode ($T_{ad} = 1$); 2, slow mode ($T_{ad} = 1$); 3, fast mode ($T_{ad} = 1$); 4, center mode, ($T_{ad} = 4$); 5, slow mode, ($T_{ad} = 4$); and 6, fast mode, ($T_{ad} = 4$).

both nonreacting ($T_{ad} = 1$) and reacting ($T_{ad} = 4$) flows. The c_u curve represents the phase speed of a disturbance that is sonic with respect to the upper stream, and the c_l curve represents the corresponding speed with respect to the lower stream. When the relative Mach number is subsonic ($|M_{r1,2}| < 1$, region I), the nonreacting flow has only center modes (Kelvin-Helmholtz modes) that travel at the average speed of two streams. As the Mach number increases, the center modes become supersonic with respect to both streams ($|M_{r1,2}| > 1$, region III); they also become less unstable and, at high enough Mach number, they become stable. At supersonic relative Mach numbers, unstable modes that are inde-

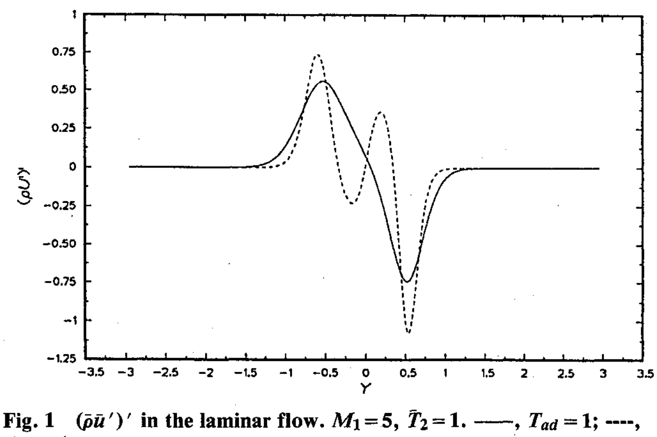

Fig. 1 $(\bar{\rho}\bar{u}')'$ in the laminar flow. $M_1 = 5$, $\bar{T}_2 = 1$. ——, $T_{ad} = 1$; ----, $T_{ad} = 4$.

pendent of the generalized inflection points arise as shown in Fig. 2. For the nonreacting flow, Fig. 3 shows that the fast mode is subsonic with respect to the upper stream and supersonic with respect to the lower stream ($|M_{r1}| < 1$, $|M_{r2}| > 1$, region II) and the slow mode is supersonic with respect to the upper stream and subsonic with respect to the lower stream ($|M_{r1}| > 1$, $|M_{r2}| < 1$, region IV). The phase speeds of the outer modes approach the freestream velocities as the Mach number increases. For the reacting flow with $T_{ad} = 4$, three inflectional unstable modes exist at low Mach numbers. As the Mach number increases, the phase speeds of the outer modes become supersonic. The center mode also becomes supersonic at high Mach numbers. The outer modes at high Mach numbers are continuations of the inflectional modes of low-Mach-number flows.

Now we study the effect of the Mach number on the maximum growth rates of two-dimensional modes. Figure 4 gives results for $T_{ad} = 1$ and 4. Only the slow mode growth rates are given for readability; the fast mode growth rates are almost identical. First consider the unheated flow ($T_{ad} = 1$). As has been demonstrated by Groppengiesser[1] and others,[2,4] the growth rate of the center mode decreases dramatically as the Mach number increases. In the high supersonic regime, the growth rate is miniscule compared to its incompressible value. At supersonic speeds, a second set of modes (outer modes) becomes unstable. The growth rate of these modes are relatively insensitive to the Mach number and, for a sufficiently high Mach number, are much larger than the growth rate of the center mode. However, they are small compared to the growth rate of the center mode at low speeds. The outer modes should be the predominant instabilities at supersonic Mach numbers.

Next, consider the reacting flow ($T_{ad} = 4$). In low-speed reacting flows,[7] the low density in the center of the shear layer reduces the growth rate of the center mode. Figure 4 shows that the growth rate of this mode is further reduced as the Mach number increases and is always smaller than the growth rate of the corresponding cold flow mode. When T_{ad} is large enough, outer modes arise in the low-speed reacting flow even at low Mach numbers. For the case shown here ($T_{ad} = 4$), the outer modes are slightly less unstable than the center mode at $M_1 = 0$; this situation reverses at higher heat release. The growth rate of the outer modes falls off much more slowly than that of the center mode with increasing Mach number and, for the heat release used here, they dominate for $M_1 > 2.5$. At $M_1 > 4$, the center mode is stable.

Figure 5 shows the maximum growth rates as functions of adiabatic flame temperature for a high-speed flow ($M_1 = 5$) as well as a lower-speed flow ($M_1 = 1$). At $M_1 = 1$, as heat release increases, the maximum amplification rate of the center mode

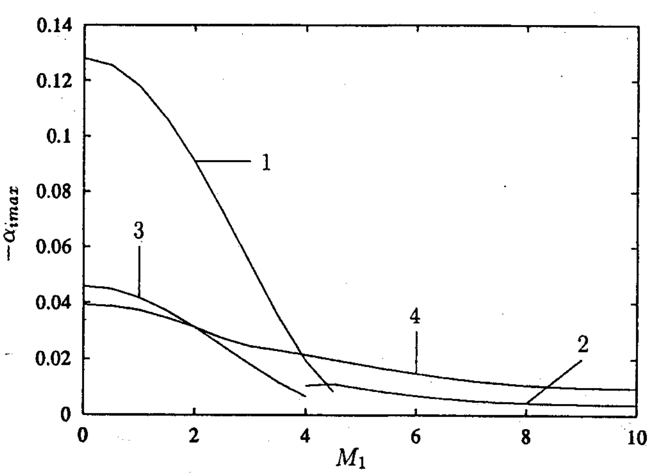

Fig. 4 Maximum growth rates vs Mach number. $\bar{T}_2 = 1$, $\beta = 0$. 1, center mode, ($T_{ad} = 1$); 2, slow mode ($T_{ad} = 1$); 3, center mode, ($T_{ad} = 4$); and 4, slow mode ($T_{ad} = 4$).

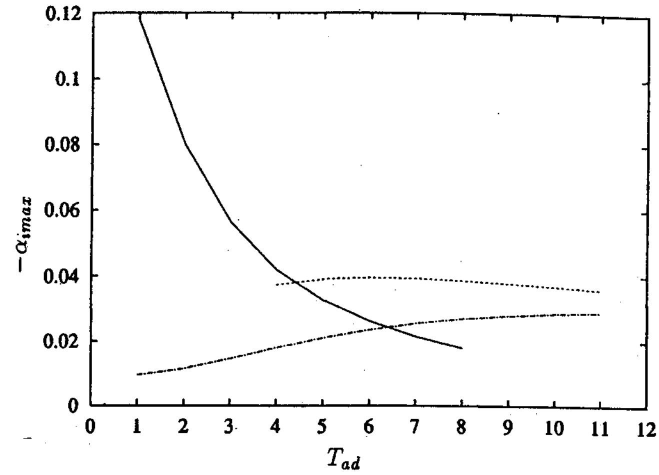

Fig. 5 Maximum growth rate vs adiabatic flame temperature. $\bar{T}_2 = 1$, $\beta = 0$. ——, $M_1 = 1$ (center mode); · · · · ·, $M_1 = 1$ (slow mode); — · —, $M_1 = 5$ (slow mode).

decreases rapidly; its value in the cold flow is 0.118, while for $T_{ad} = 8$ it is 0.018, or 15% of the cold flow value. The amplification rates of the outer modes change very little as heat release increases. Consequently, at high heat release, the outer modes have larger amplification rates than the center mode. For $T_{ad} = 8$, the outer modes have almost twice the amplification rate of the center mode. Flows with high heat release ($T_{ad} > 5$) should be unstable to the outer mode, but heat release stabilizes the flow at $M_1 = 1$ ($M_c = 0.25$).

At $M_1 = 5$ ($M_c = 1.25$), only the supersonic outer modes are unstable. As heat release increases, their maximum growth rates increase. The maximum growth rate of the slow outer mode of the nonreacting flow is 0.0095, while for $T_{ad} = 8$ it is 0.027, or about three times as large. Reacting experiments of Hall[17] showed that the shear-layer thickness increased by 10% with increasing heat release ($T_{ad} \simeq 2$). According to our calculations with the same T_{ad}, the growth rate increases by about 20%. Thus, there is just a small qualitative difference between our predictions and his experiments. This might be due to different Reynolds numbers and/or initial profiles. We take this as evidence that heat release destabilizes the supersonic flow in contrast to the low-speed case.

C. Three Dimensionality

Sandham and Reynolds[2] reported that the most unstable mode in compressible flows becomes three-dimensional when $M_c > 0.6$. In this section, we study the effect of heat release ($T_{ad} = 4$) on the obliquity of the most amplified mode in high-speed flows ($M_1 = 5$).

Figure 6 shows the maximum growth rates and phase speeds for both nonreacting ($T_{ad} = 1$) and reacting ($T_{ad} = 4$) flows. Only the slow mode growth rates are given for readability; the fast mode growth rates are almost same. In a nonreacting mixing layer, for angles less than about 37 deg, the outer modes are dominant and the maximum growth rates change little with angle. They are supersonic unstable modes that have no connection with the generalized inflection points. However, when the angle is greater than 37 deg, the outer modes disappear and the center mode begins to dominate. The reason for the transition from outer mode dominance to center mode dominance can be understood by examining the relative Mach number, M_r [Eq. (8)]. For waves propagating at angle θ relative to the x direction, Eqs. (8) and (9) become

$$M_{r1} = M_1(c - 1)\cos\theta, \qquad M_{r2} = \frac{M_1(c - \bar{u}_2)\cos\theta}{\bar{T}_2^{1/2}} \tag{13}$$

$$c_u = 1 - \frac{1}{M_1 \cos\theta}, \qquad c_l = \bar{u}_2 + \frac{\bar{T}_2^{1/2}}{M_1 \cos\theta} \tag{14}$$

where c is the phase velocity, ω/α. $c_{u,l}$ are the phase speeds of disturbances that are sonic with respect to the upper and lower streams, respectively ($|M_{r1,2}| = 1$). $c_{u,l}$ are plotted in Fig. 6b for $M_1 = 5$. Because the relative Mach numbers are functions of the propagation angle, there is a possibility of transition from supersonic (regions II, III, and IV) to subsonic (region I) disturbances as the angle increases. $c_u = c_l$ defines the smallest transition angle and, for $M_1 = 5$, it is about 37 deg. The center modes in Fig. 6b above 37 deg are subsonic relative to both free streams.

Sandham and Reynolds[2] found that, for the most amplified disturbance,

$$M_c \cos\theta \approx 0.6 \tag{15}$$

This relation predicts the most unstable mode to be at 62 deg for $M_c = 1.25$ ($M_1 = 5$), which is very close to the angle of maximum instability of the center modes (65 deg). The relative Mach number for the most unstable center mode is 0.53 and is subsonic. Note that the maximum growth rate of oblique center modes is much greater than the growth rate of two-dimensional outer modes. Therefore, the most unstable mode is oblique and subsonic for nonreacting flow at $M_1 = 5$.

At $T_{ad} = 4$, even though the relative Mach number becomes subsonic with increasing obliquity, the center mode is stabilized by the heat release and only the outer modes are amplified. The latter are inflectional modes whose maximum amplification rates decrease as the obliquity increases. Therefore heat release makes the dominant mode two-dimensional even in the high-Mach-number regime. The three-dimensional modes which dominate in the nonreacting case are stable and Eq. (15) is apparently not applicable in reacting mixing layers.

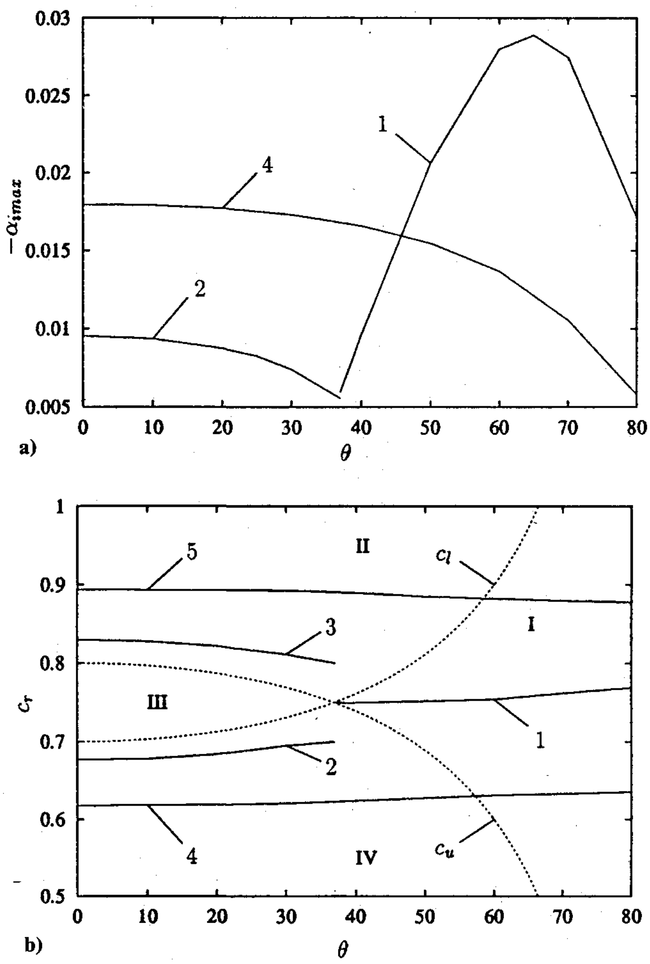

Fig. 6 a) Maximum growth rate vs oblique angle and b) corresponding phase velocity. $M_1 = 5$ ($M_c = 1.25$), $\bar{T}_2 = 1$. 1, $T_{ad} = 1$ (center mode); 2, $T_{ad} = 1$ (slow mode); 3, $T_{ad} = 1$ (fast mode); 4, $T_{ad} = 4$ (slow mode); and 5, $T_{ad} = 4$ (fast mode).

D. Effect of Damköhler Number and Equivalence Ratio

To see the effect of the Damköhler number Da on the growth rates of reacting mixing layers, we studied the maximum growth rates of the unstable modes as functions of Damköhler number. Here, the Damköhler number is defined as the ratio of the convective flow time to the chemical reaction time in the laminar flow

$$Da = \frac{\delta\omega}{\bar{u}_1} \Big/ \left[\frac{W_F \nu_F}{W_F^{\nu F} W_O^{\nu O}} B \bar{\rho}_1^{(\nu F + \nu O - 1)} \exp\left(-\frac{E}{R\bar{T}_1} \right) \right]^{-1} \tag{16}$$

where δ_ω is the initial vorticity thickness, $W_{F,O}$ the molecular weight, and $\nu_{F,O}$ the stoichiometric coefficients of the fuel and oxidizer, respectively. The quantity E is the activation energy of the reaction, B is the frequency factor, and R is the universal gas constant.

Figure 7a shows the temperature profiles with the same inlet profiles and different Damköhler numbers at $M_1 = 5$ and $T_{ad} = 4$. These profiles are compared at a particular nondimensional downstream distance x. As expected, more heat is released as Da increases. At low Damköhler numbers, the heat release increases rapidly with the speed of the chemical reac-

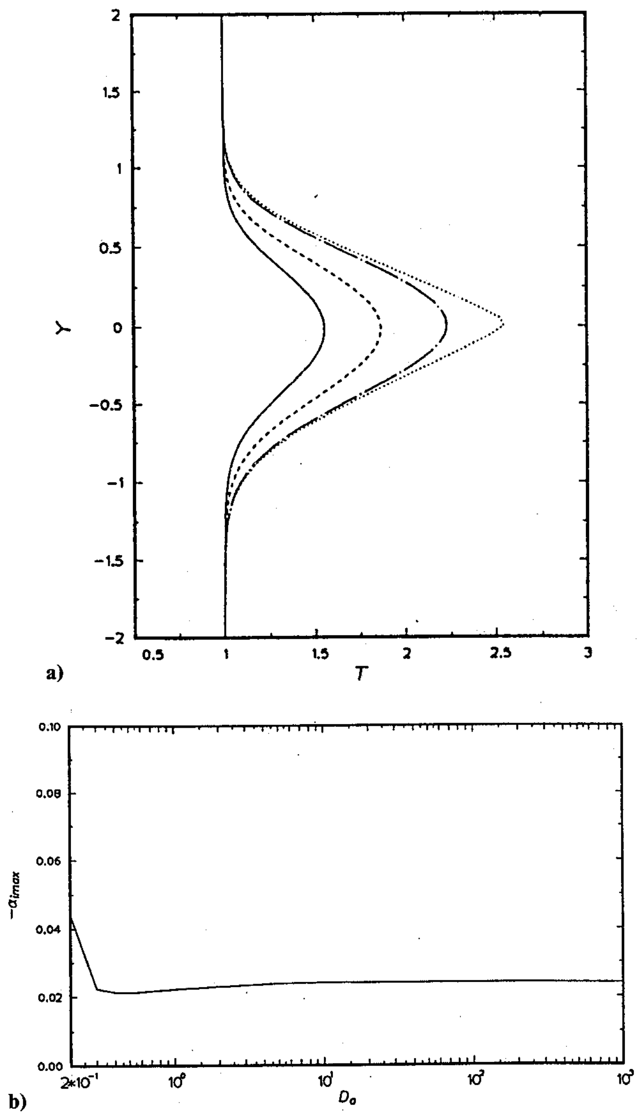

Fig. 7 Effect of Damköhler number. $M_1 = 5$, $\bar{T}_2 = 1$, $T_{ad} = 4$, $\beta = 0$: a) temperature, ——, $Da = 0.2$; ----, $Da = 0.5$; — - —, $Da = 2$; · · · · ·, $Da = 1000$; and b) maximum growth rate.

tion and the behavior is said to be reaction limited. At high Damköhler numbers, heat release increases little and the temperature profile is nearby independent of Da; the growth is diffusion limited. The maximum growth rates shown in Fig. 7b decrease considerably until $Da \simeq 0.5$, after which they rapidly approach their asymptotic high Da value. Increasing Damköhler number stabilizes the flow at low Damköhler numbers, but has little effect (it is slightly destabilizing) at high Damköhler numbers. These results show that the flamesheet model is valid for $Da \geq 0.5$ but finite rate chemistry needs to be considered at low Damköhler numbers ($Da < 0.5$).

The equivalence ratio of a diffusion flame can be defined by

$$\phi = \frac{(Y_F/Y_O)_{\text{real}}}{(Y_F/Y_O)_{\text{ideal}}} \tag{17}$$

where Y_F, Y_O represents the mass fractions of the fuel oxidizer in the freestreams. We assume that $(Y_F/Y_O)_{\text{ideal}} = 1$. If $\phi > 1$, the mixture is fuel rich, while if $\phi < 1$ it is fuel lean. Here we study the effect of equivalence ratio on the stability of high-speed reacting flows; three equivalence ratios, $\phi = 0.5$, 1, 2, are used at $T_{ad} = 4$ and $Da = 10$. The upper stream contains the fuel, and the lower stream, the oxidizer. Figure 8 shows the maximum growth rates as functions of the Mach number for various equivalence ratios. The effect of equivalence ratio is large at low Mach numbers. Because any deviation from stoichiometric conditions reduces the total heat release, the stoichoimetric case ($\phi = 1$) has a lower growth rate than the others ($\phi = 0.5$, 2). The fuel-lean case ($\phi = 0.5$) is the most unstable one up to $M_1 = 3.5$. At high Mach numbers, the growth rate for the stoichiometric case is larger than the others because heat release now destabilizes the flow (see Sec. IIIB). However, the change in growth rates at high Mach numbers is smaller than at low Mach numbers.

E. Contours

From the eigenfunctions of the most unstable modes and the mean flow, a qualitative approximation to a typical flow variable can be calculated. We plotted selected contours of the flow variables. Only the slow supersonic mode is shown; the fast supersonic mode can be obtained by reflection. Because the action of the slow mode is concentrated in the lower part of the mixing layer, it disturbs this region. Figure 9 shows the contours of the flow variables produced by the supersonic slow mode for the nonreacting ($T_{ad} = 1$) flow at $M_1 = 5$ ($M_c = 1.25$). The vorticity divided by density in Fig. 9a is quite different from the incompressible flow structure;[8] it shows a single clockwise core on the slow side of the layer. Sandham and Reynolds[18] used the compressible vorticity equations to explain the reduced growth rate in compressible mixing layers. Along these lines, we checked the dilatational and baroclinic terms. In the region near the vortex, both the dilational and baroclinic terms are negative, and may act to reduce the

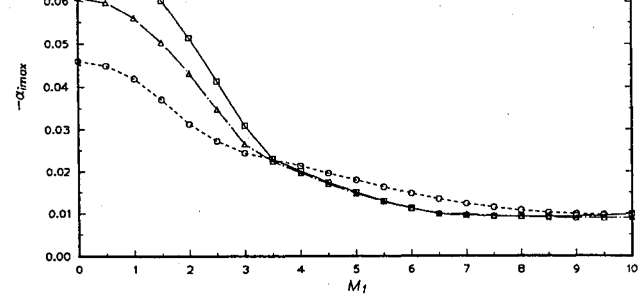

Fig. 8 Effect of equivalence ratio on maximum growth rate. $T_{ad} = 4$, $\bar{T}_2 = 1$, $\beta = 1$. □, $\phi = 0.5$; ○, $\phi_2 = 1$; △, $\phi = 2$.

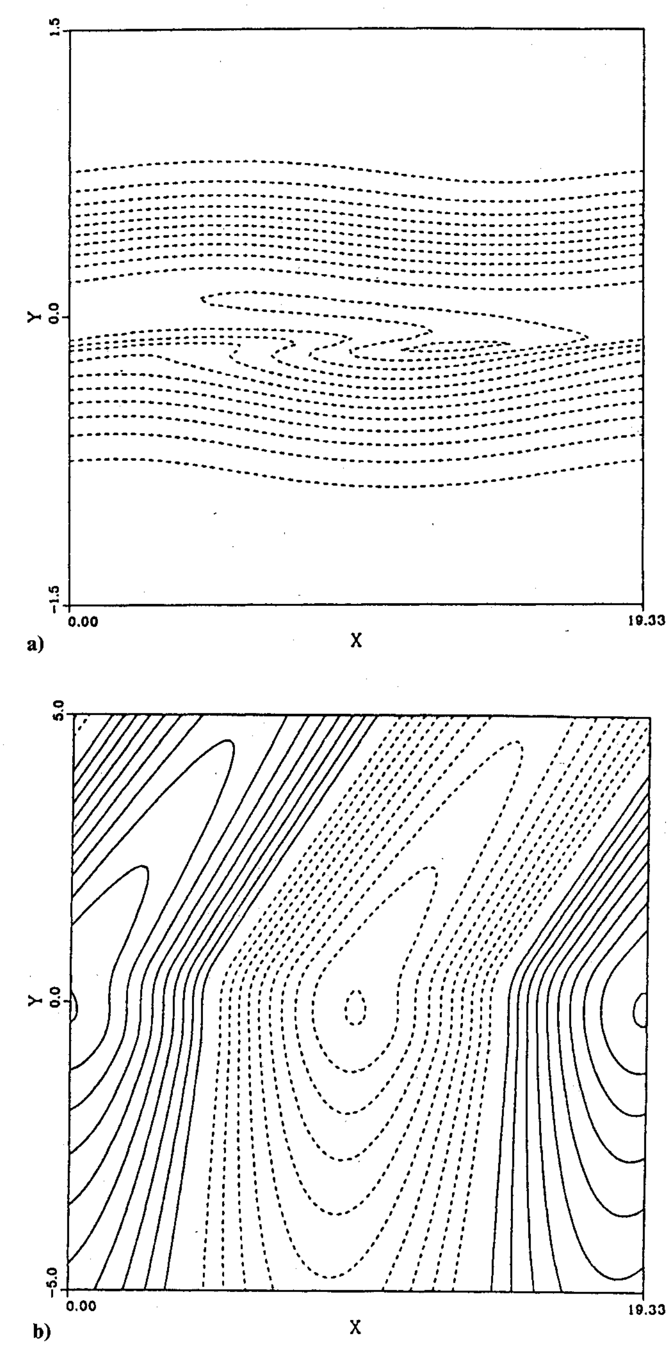

Fig. 9 Contour plots from linear eigenfunctions of the nonreacting flow (slow mode). $M_1 = 5$ ($M_c = 1.25$), $\bar{T}_2 = 1$, $T_{ad} = 1$, $\beta = 0$: a) vorticity/density (max $= 8.58 \times 10^{-4}$, min $= -0.367$); and b) pressure (max $= 1.05$, min $= 0.95$).

growth of the two-dimensional instability. The pressure contours in Fig. 9b show clearly the radiative nature of the supersonic mode. On the lower side of the layer, which is subsonic relative to the disturbance, the pressure distribution is similar to what is found in an incompressible flow. However, the upper side, which is supersonic relative to the disturbance, shows compression (solid lines) and expansion (dashed lines) waves propagating to infinity. The radiation of wave energy to infinity is probably a major cause of the decreased growth rate.

Figure 10 gives contours produced by the supersonic slow mode for the reacting flow ($T_{ad} = 4$) at $M_1 = 5$ ($M_c = 1.25$). Figure 10a shows that the extrema of the vorticity divided by density lie below the center of the layer. The dilatational and baroclinic terms are both negative near the vortex, and tend to inhibit the growth of the two-dimensional instability. The

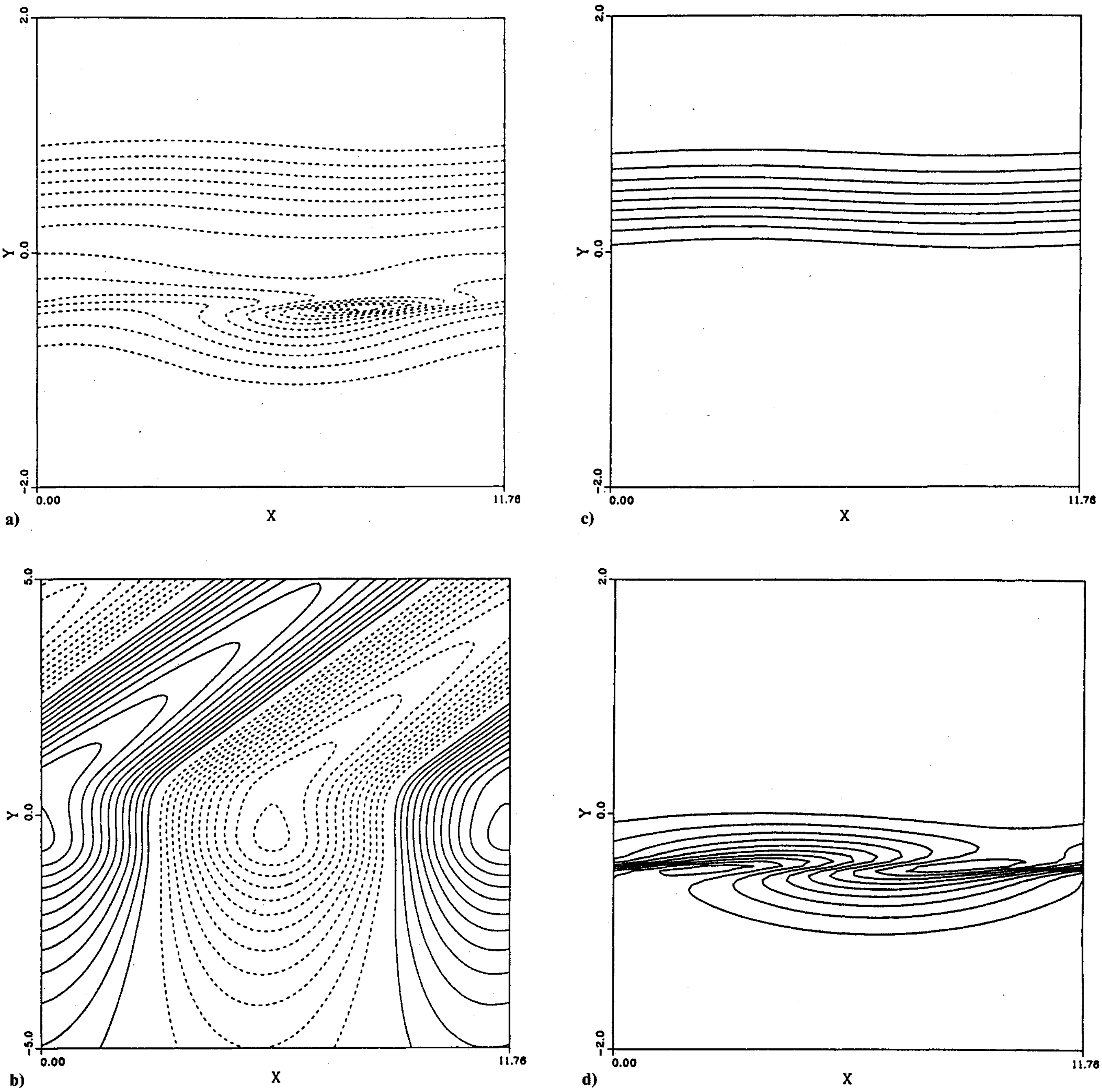

Fig. 10 Contour plots from linear eigenfunctions of the reacting flow (slow mode). $M_1 = 5$ ($M_c = 1.25$), $\bar{T}_2 = 1$, $T_{ad} = 4$, $\beta = 0$, $Da = 10$: a) vorticity/density (max = 2.34×10^{-4}, min = -0.564); b) pressure (max = 1.062, min = 0.938); c) fuel (max = 1.0, min = 0.0); and d) oxidizer (max = 1.0, min = 0.0).

pressure contours in Fig. 10b exhibit the radiation of compression (solid line) and expansion (dashed line) waves on the upper side. Figures 10c–d show the mass fractions of the reactants. Because the fuel occupies the upper part of the layer and the oxidizer the lower part, the slow mode principally affects the oxidizer. The fuel distribution is hardly perturbed and the slow mode does not increase mixing between the reactants very much.

In Sec. IIIC, we showed that the most unstable mode in a $M_1 = 5$ ($M_c = 1.25$) nonreacting flow is a center mode with oblique angle $\theta = 65$ deg and that it is subsonic relative to both freestreams. To check the subsonic nature of the oblique center mode, we plot contours of pressure produced by this mode in Fig. 11. The pressure contours are similar to the ones of the subsonic flow and show no propagation of pressure waves toward the boundaries. This flow is similar to a subsonic flow in many ways.

F. Convective Mach Number

Use of the convective Mach number has been suggested as a way to collapse growth-rate data onto one curve.[14,19,20] The best known definition of M_c is based on the velocity of a frame convecting with large structures of the mixing layer.[14] By assuming the existence of a stable stagnation point of the kind found in incompressible flows, that the dynamic pressures on the two sides of the stagnation point are equal and that the process is isentropic, Papamoschou and Roshko[14] derived an expression for the convective Mach number

$$M_c = M_{c1} = M_{c2} \frac{M_1(1 - \bar{u}_2)}{1 + \sqrt{\bar{T}_2}} \tag{18}$$

Papamoschou and Roshko[14] suggested that the growth rate of a compressible shear layer normalized by the growth rate of an incompressible shear layer might be a function solely of the

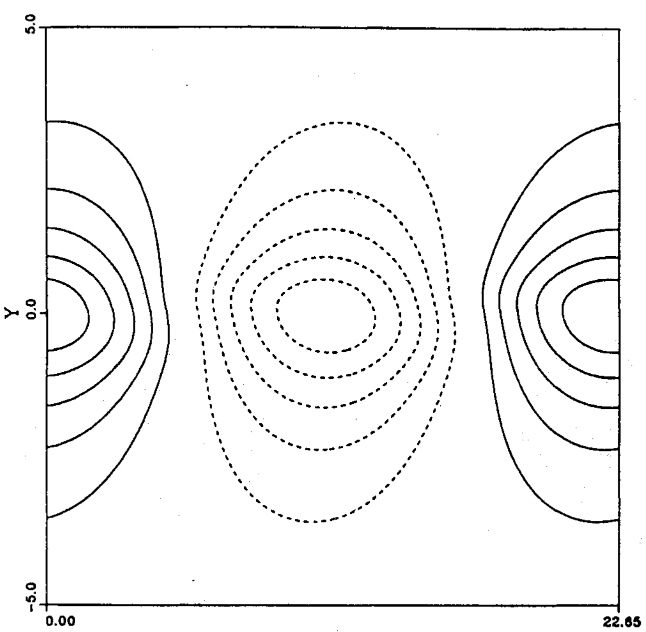

Fig. 11 Pressure contours of the most unstable oblique center mode ($\theta = 65$ deg), Max = 1.018, min = 0.982. $M_1 = 5$ ($M_c = 1.25$), $\bar{T}_2 = 1$, $T_{ad} = 1$.

convective Mach number for a wide range of velocity and temperature ratios. They showed that the normalized growth rate in turbulent flows decreases with increasing convective Mach number. Recently, Ragab and Wu[20] studied the influence of the velocity ratio on stability characteristics of a compressible shear layer and found that the isentropic convective Mach number does correlate compressibility effects on the growth rate.

A second definition takes the convective velocity to be the phase velocity of the most unstable mode according to linear-stability theory. This was proposed by Mack[9] for compressible boundary layers and later used by Zhuant et al.[19] for compressible shear layers. The convective Mach number for the two freestreams can be written as

$$M_{c1} = M_1(1 - c_r), \qquad M_{c2} = \frac{M_1(c_r - \bar{u}_2)}{\sqrt{\bar{T}_2}} \tag{19}$$

where c_r is the phase velocity of the most unstable mode. The convective Mach numbers of Eqs. (18) and (19) are identical when the dominant mode is the center mode and $\bar{T}_2 = 1$, because the large structures then move with the phase speed of the center mode. When the outer modes dominate due to heat release or compressibility, the isentropic convective velocity remains the center mode phase velocity but the convective velocity based on the most unstable mode becomes the phase velocity of the outer modes, and, therefore, convective Mach numbers from Eqs. (18) and (19) become different. Zhuang et al.,[19] who used the convective Mach number based on the most unstable mode, compared their results with those of Ragab and Wu[20] who used the isentropic convective Mach number. Even though the difference in supersonic convective Mach numbers is not small, the comparison showed good agreement in the normalized growth rates because they change little at supersonic convective Mach numbers.

To see whether convective Mach numbers can correlate shear-layer compressibility in reacting mixing layers, we normalize maximum growth rates for various temperature ratios and adiabatic flame temperatures by incompressible growth rates at the same velocity and temperature ratios and adiabatic flame temperatures to isolate the effect of compressibility:

$$R(M_c) = \frac{-\alpha_{i\max}(M_c, \bar{u}_2, \bar{T}_2, T_{ad})}{-\alpha_{i\max}(0, \bar{u}_2, \bar{T}_2, T_{ad})} \tag{20}$$

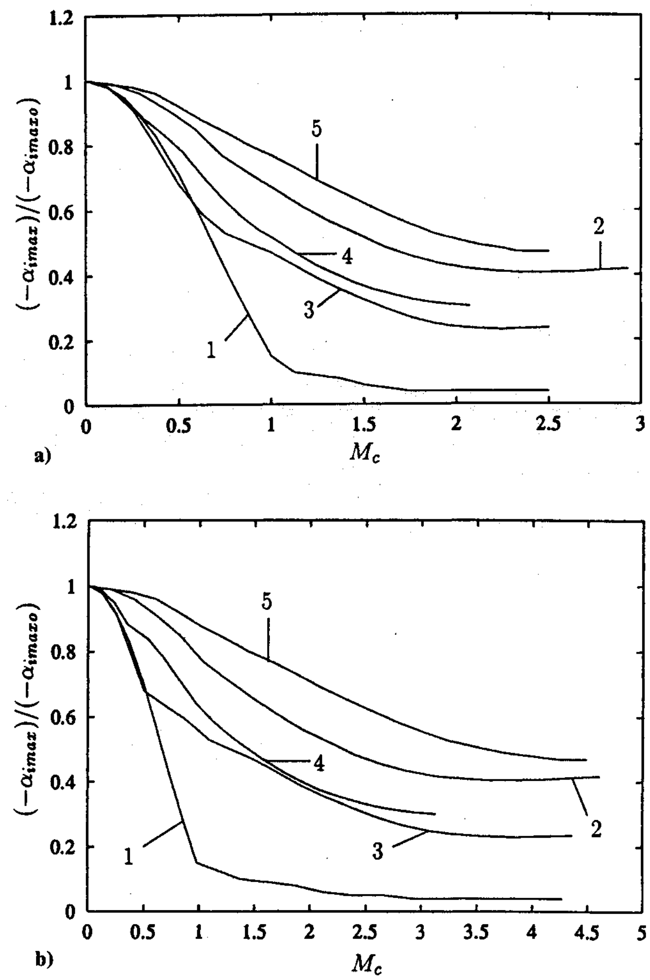

Fig. 12 Normalized maximum growth rates vs convective Mach number for reacting flows: a) isentropic convective Mach number [Eq. (17)]; and b) convective Mach number based on the most unstable mode [Eq. (18)]. $\bar{u}_2 = 0.5$, $\beta = 0$. 1, $\bar{T}_2 = 1$, $T_{ad} = 1$; 2, $\bar{T}_2 = 0.5$, $T_{ad} = 4$; 3, $\bar{T}_2 = 1$, $T_{ad} = 4$; 4, $\bar{T}_2 = 2$, $T_{ad} = 4$; 5, $\bar{T}_2 = 1$, $T_{ad} = 8$.

Papamoschou and Roshko[14] used the same normalized growth rates without the chemical reaction. Figure 12 shows the normalized maximum growth rates vs the two convective Mach numbers. The results show that the growth rates do not collapse with either convective Mach number. Thus, convective Mach number does not appear to be a universal parameter expressing shear-layer compressibility effects in this sense.

IV. Conclusions

In this work, we considered the inviscid stability of compressible reacting mixing layers. The calculations are based on laminar flow profiles generated by solving the compressible boundary-layer equations with finite-rate chemistry. We found that supersonic unstable modes may exist in the absence of a generalized inflection point, provided that a region of laminar flow is supersonic relative to the disturbance phase velocity. The growth rate decreases with increasing Mach number. At supersonic Mach numbers, the outer modes dominate. Heat release stabilizes low-speed flows but destabilizes high-speed flows. However, the growth rates are small compared to the compressible cold flow value. For nonreacting supersonic flows at $M_c > 0.6$, the most unstable modes are oblique center modes that are subsonic relative to both freestreams. For reacting flows with $T_{ad} > 3$, the most unstable modes are two-dimensional outer modes even at high Mach numbers. The radiative nature of supersonic disturbances is demonstrated by the pressure contours; the radiation of energy is one reason for the decreased growth rates. Supersonic disturbances do not mix the reactants very well because they are largely confined to one side of the flow. For reacting flows, the growth rates normalized by the corresponding incompressible growth rates are not functions of the convective Mach number alone, so the latter cannot be used as an overall measure of shear-layer compressibility.

Appendix: Representation of [RXN1] and [RXN2]

The terms [RXN1] and [RXN2] of Eq. (4) may be written as ratios of determinants:

$$[RXN1] = \frac{\begin{vmatrix} A & C & B \\ H & F & G \\ N & M & -L \end{vmatrix}}{\begin{vmatrix} A & C & -D \\ H & F & -I \\ N & M & -K \end{vmatrix}}, \quad [RXN2] = \frac{\begin{vmatrix} A & C & E \\ H & F & J \\ N & M & O \end{vmatrix}}{\begin{vmatrix} A & C & -D \\ H & F & -I \\ N & M & -K \end{vmatrix}}$$

The elements of these determinants are

$$A = i\bar{\rho}((\alpha\bar{u} - \omega) + Da\,\bar{\rho}^2\bar{y}_O \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$B = \frac{-i\bar{y}_F'}{(\alpha\bar{u} - \omega)}$$

$$C = Da\,\bar{\rho}^2\bar{y}_F \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$D = Da\left[\frac{\bar{\rho}^2}{\bar{T}^2}\bar{y}_F\bar{y}_O\beta' - 2\frac{\bar{\rho}^2}{\bar{T}}\bar{y}_F\bar{y}_O\right]\exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$E = 2Da\,\bar{\rho}^2\bar{y}_F\bar{y}_O \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$F = i\bar{\rho}(\alpha\bar{u} - \omega) + Da\,\frac{W_O\nu_O}{W_F\nu_F}\bar{\rho}^2\bar{y}_O \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$G = \frac{-i\bar{y}_O'}{(\alpha\bar{u} - \omega)}$$

$$H = Da\,\frac{W_O\nu_O}{W_F\nu_F}\bar{\rho}^2\bar{y}_O \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$I = Da\frac{W_O\nu_O}{W_F\nu_F}\left[\frac{\bar{\rho}^2}{\bar{T}^2}\bar{y}_F\bar{y}_O\beta' - 2\frac{\bar{\rho}^2}{\bar{T}}\bar{y}_F\bar{y}_O\right]\exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right.$$

$$J = E$$

$$K = i\gamma\bar{\rho}(\alpha\bar{u} - \omega) - Da\,\frac{Q}{W_F\nu_F}\left[\frac{\bar{\rho}^2}{\bar{T}^2}\bar{y}_F\bar{y}_O\beta' - 2\frac{\bar{\rho}^2}{\bar{T}}\bar{y}_F\bar{y}_O\right]$$
$$\exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$L = -\frac{-i\bar{T}'}{(\alpha\bar{u} - \omega)} + \frac{-i\rho(\gamma - 1)}{\bar{\rho}^2(\alpha\bar{u} - \omega)}$$

$$M = Da\,\frac{Q}{W_F\nu_F}\bar{\rho}^2\bar{y}_F \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$N = Da\,\frac{Q}{W_F\nu_F}\bar{\rho}^2\bar{y}_O \exp\left[-\beta'\left(\frac{1}{\bar{T}} - 1\right)\right]$$

$$O = i(\gamma - 1)\,(\alpha\bar{u} - \omega) + 2Da\frac{Q}{W_F\nu_F}\,\bar{\rho}^2\bar{y}_F\bar{y}_O$$
$$\exp\left[-\beta'\left(\frac{1}{\bar{T} - 1}\right)\right]$$

where β' and Q represent the nondimensional parameters of activation energy and heat release, respectively. The terms W and ν represent, respectively, the molecular weight and stoichiometric coefficient of species.

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